647510 **BSc6MatC602x** **Seat No :________**

B.Sc. Semester - 6 (*CBCS*) Examination
March/April-2026 (NEW COURSE)
Mathematical Analysis - 2 and Abstract Algebra-2 (Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Attempt any two out of three (10)

(1) Prove that open subset of metric space is not compact.
(2) Determine whether $\mathbb{Z}$ is compact, connected or denumerable set.
(3) State and prove theorem of Nested Intervals.

Que.1(B) Answer any one out of two (04)

(1) Prove that intersection of two compact sets is a compact.
(2) Prove or disprove any two disjoint subsets of a metric space are separated.

Que.2(A) Attempt any two out of three (10)

(1) State and prove change of scale property of Laplace transform and first shifting property of Laplace transform.
(2) Find $L(e^{2t}sin^4t)$ and $L(t^2e^{2t})$.
(3) Find $L^{-1}\left(\frac{2s^2-6s+5}{(s^3-6s^2+11s-6)}\right)$ and $L^{-1}\left(\frac{5s+3}{(s-1)(s^2+2s+5)}\right)$.

Que.2(B) Answer any one out of two (04)

(1) Find $L(f(t),\ where\ f(t) = \begin{cases} \sin\left(t-\frac{\pi}{3}\right), & \pi/3 < t \\ 0, & 0 < t \le \pi/3 \end{cases}$
(2) Find $L(\ e^{2t}(3t^5 - \cos 4t\ )\ )$.

Que.3(A) Attempt any two out of three (10)

(1) Derive the formula for integration of Laplace transform.
(2) State and prove convolution theorem.
(3) find $L\left(\frac{cosat-cosbt}{t} + tsinat\right)$ and $L(\ te^{2t}sin3t\ )$.

Que.3(B) Answer any one out of two (04)

(1) Using convolution theorem, find $L^{-1}(\frac{1}{(s^2+4s+13)^2})$.
(2) Solve $y''' + 2y'' - y' - 2y = 0,\ given\ that$ $y(0) = y'(0) = 0\ and\ y''(0) = 6$ by using Laplace transform.

Que.4(A) Attempt any two out of three (10)

(1) If $\varphi: (G,*) \to (G', \Delta)$ be an into homomorphism and if K_φ is the kernel of φ then $G/K_\varphi \cong \varphi(G)$.
(2) Let $I_1\ and\ I_2$ be any ideals of a ring R. Then, prove that $I_1 \cup I_2$ is also an ideal of R iff either $I_1 \subseteq I_2\ or\ I_2 \subseteq I_1$.
(3) A commutative ring with unity is a field if it has no proper ideal.

Que.4(B) Answer any one out of two (04)

(1) Show that it is impossible to find a homomorphism from $(\mathbb{Z}_5, +_5)$ onto $(\mathbb{Z}_4, +_4)$.
(2) Show that $\varphi: G \to G; \varphi(x) = x^3, \forall x \in G$ is a homomorphism where $G = (C - \{0\},\cdot)$ with $K_\varphi = \{1, \omega, \omega^2\}$ where ω is a cube root of unity.

Que.5(A) Attempt any two out of three (10)

(1) Define a polynomial. Find $f + g\ and\ fg$ for the polynomials $f = (2,0,-3,0,4,0,0, \dots)\ and\ g = (1,-2,0,0, \dots) \in \mathbb{R}[x]$.
(2) State and prove Remainder theorem.
(3) State and prove Division algorithm for polynomials

Que.5(B) Answer any one out of two (04)

(1) Define irreducible and reducible polynomials. Give an example of polynomial which is reducible over $\mathbb{R}$ but not over $\mathbb{Q}$.
(2) Find g.c.d. of $f(x) = x^3 + 3x^2 + 3x + 3$ and $g(x) = 4x^3 + 2x^2 + 2x + 2 \in \mathbb{Z}_5[x]$.

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